

The Careless Stalemate

Summer 2009

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1 Puzzle

A chess puzzle this week. A rather exciting game has just ended in a surprising stalemate. Despite Black only having their king remaining, and White possessing exactly one of each piece (1 King, 1 Queen, 1 Rook etc), Black managed to squeak in with a stalemate.

The stalemate was interesting not just because of the pieces involved. All of White's pieces were involved in the stalemate, and none of the 8 squares around the Black king was guarded twice. Also, all of them were very specifically placed - moving any of them elsewhere on the board would no longer satisfy that condition.

Given that the Kings were in the position shown in Figure 1, were the other pieces all were?

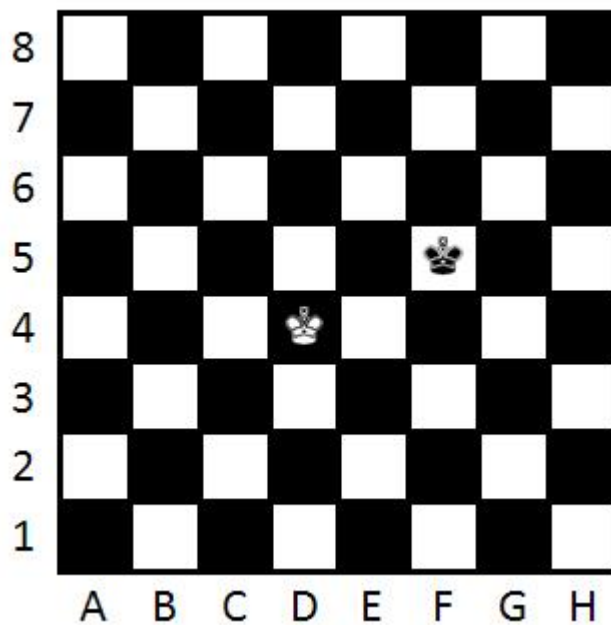


Figure 1: Positions of the kings in the stalemate

2 Discussion

First off, a clarification. When we say all of White's pieces were involved in the stalemate, this means that removing any of them from the board would mean it was no longer a stalemate. And the statement that none of the 8 squares around the Black king was guarded twice means exactly that - no White piece is guarding a square already guarded by another piece.

That alone puts a lot of restraint on where the White pieces can go. The final condition - that moving one piece invalidates it - means that you can't have one piece free to move along a line of squares. Another way to see this would be to list all the possible solutions that satisfy the first two conditions and then find one which couldn't be turned into another by only moving one piece.

3 Solution

There is a huge amount of logic deduction that goes on here - arguments along the line of 'assume this piece goes here, then that has to go there, and then that has to go there, so that doesn't work, so the first piece has to go somewhere else'. For illustration I will include one line of the logic below, but then skip straight to the answer. If anyone does want the full version, feel free to email me, my details are on the site.

First off, we note that we have 5 remaining pieces (Queen, Rook, Knight, Bishop and Pawn) and 6 squares left to cover as the White King is covering 2. So now piece can cover 3 squares, or else we have only 3 squares left for the other 4 pieces. Specifically the rook cannot cover 3 squares, so he must have the line he is covering blocked by another piece. Thus at least one of the 8 squares must be occupied by one of the other 4 pieces.

It is immediately obvious that it cannot be occupied by the Queen, or else the Black King would be in check, invalidating the stalemate. Also the occupying piece cannot be on e4 or e5, the two squares covered by the White King - then it would be covered by both the Rook and the White King, meaning these squares were doubly covered, which is invalid.

Try the assumption that it is the Knight that blocks the Rook. Because the Knight cannot also cover e4 or e5 it can only go on f4 or e6. First, assume it is on f4. This leaves us with 3 squares left to cover with the other 3 pieces. 2 are black, and 1 is white - so at least 1 black must be covered by a Bishop/Queen (it is immediately obvious that to avoid the same problem with the Queen needing blocking it will cover squares on the diagonal not on straight lines). f6 cannot be covered by a bishop - if it is occupied then it must be so by the pawn, which therefore doesn't contribute, and if it is unoccupied then covering f6 also covers g5, leaving 2 pieces to cover the remaining square of g4, which is a problem.

So we can assume that g5 is covered without covering either f4 or f6. Thus g5 must be occupied, and so it must be occupied by the Pawn, as the Queen could not be there. It must be protected therefore by a Bishop on h4 or h6, leaving the Queen to cover square g5 from d1, the only place it can do so from without interfering. We thus gain our first solution which is

Qd1, Rf1/f2, Knf4, Pg5, Bh4/h6

That reasoning should be sufficient to give an idea of the logic you have to employ. You end up getting the following set of solutions.

Q d1, R f1/f2, Kn f4, B h4/h6, P g5

Q d1, R a6/b6/c6/d6/e7/e8, Kn e6, B e8, P g5/e5

Q a2/b3/c4, R h5, Kn h8, B g5, P f3/h3

Q d1, R h5, Kn h8, B g5, P d5 *

Q a2/b3/c4, R g1/g2/g3, Kn e2, B e8/f7, P g5

Q a2/b3/c4, R g3, Kn e2/g2, B e8/f7, P g5

Q d1, R g7, Kn g2, B a2/b3/c4, P g5

Q a2/b3/c4, R g7, Kn g2, B d1/e2, P g5

Clearly we can make out the one which is unique, the fourth line (marked *), doesn't have another solution which can be formed by only moving one piece. So that is the solution, as shown in figure 2.

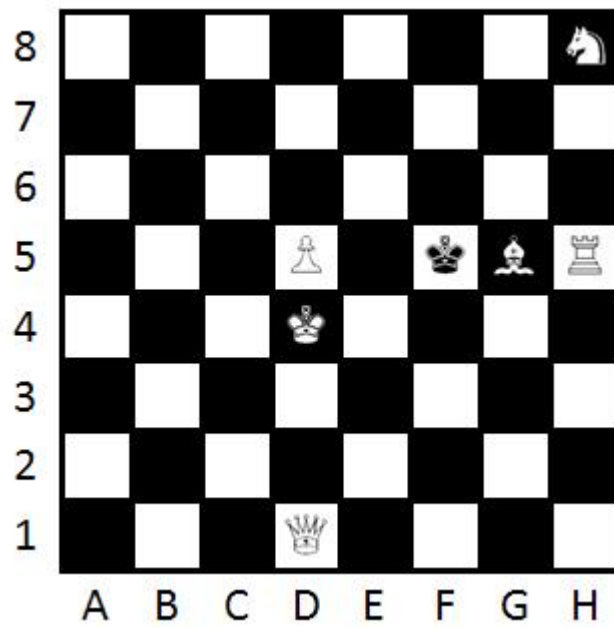


Figure 2: Solution to the Stalemate Problem